STATISTICS IN TRANSITION-new series, October 2009
Vol. 10, No. 2, pp. 223—233

MULTISTAGE BALANCED GROUPS RANKED SET SAMPLES FOR ESTIMATING THE POPULATION MEDIAN

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ABSTRACT

A multistage balanced groups ranked set samples (MBGRSS) method and its properties for estimating the population median is considered. The suggested estimator is compared to those obtained based on simple random sampling (SRS) and the ranked set sampling (RSS) methods. The MBGRSS estimator of the population median is found to be unbiased if the underlying distribution is symmetric and has a small bias if the underlying distribution is asymmetric, the bias is decreasing in r (r is the number of stage). It is found that, MBGRSS is as efficient as RSS when $m = 3$ and $r = 1$, and it is more efficient than RSS for $r > 1$. However, the efficiency of MBGRSS is increasing in r for specific value of the sample size whether the underlying distribution is symmetric or asymmetric. Real data is used to illustrate the method.

Key words: Ranked set sampling; simple random sampling; balanced groups ranked set samples, symmetric distribution; asymmetric distribution.

1. Introduction

McIntyre (1952) introduced the ranked set sampling method to estimate mean pasture and forage yields. Takahasi and Wakimoto (1968) provided the necessary mathematical theory of RSS. Dell and Clutter (1972) considered the case in which the ranking may be done with errors. Samawi et al. (1996) investigated the extreme ranked set samples (ERSS) for estimating a population mean. Muttlak (1997) suggested using median ranked set sampling (MRSS) to estimate the population mean. Al-Saleh and Al-Omari (2002) proposed the multistage ranked set sampling (MSRSS) method to increase the efficiency when estimating the

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population mean for specific value of the sample size. Muttlak (2003a, 2003b) suggested quartile ranked set sampling (QRSS) and percentile ranked set sampling (PRSS) for estimating the population mean and showed that PRSS and QRSS produced unbiased estimators of the population mean when the underlying distribution is symmetric. Jemain and Al-Omari (2006) proposed multistage median ranked set sampling (MMRSS) method for estimating the population mean. They found that MMRSS is more efficient than the commonly used SRS based on the same sample size. Jemain et al. (2007) suggested multistage extreme ranked set sampling (MERSS) method for estimating the population mean. Al-Omari and Jaber (2008) investigated estimation the population mean using percentile double ranked set sampling. For more details about RSS see Zheng and Modarres (2006), Tseng and Shao (2007), and Ozturk and Deshpande (2006), and Tiensuwan et al., (2007)

1.1.Ranked set sampling

The RSS involves randomly selecting m^2 units from the population. These units are randomly allocated into m sets, each of size m. The m units of each sample are ranked visually or by any inexpensive method with respect to the variable of interest. From the first set of m units, the smallest unit is measured. From the second set of m units, the second smallest unit is measured. The process is continued until from the mth set of m units the largest unit is measured. Repeat the process n times to have a set of size mn from the initial m^2n units.

1.2.Multistage balanced groups ranked set samples

The MBGRSS procedure consists of the following steps:

Step 1: Randomly select $(3k)^{r+1}$ $(k = 1, 2, ...)$ units from the target population. These units are randomly allocated into $(3k)^r$ sets, each of size $3k$.

Step 2: The $3k$ units of each set are ranked visually or by any inexpensive method with respect to the variable of interest. Then the $(3k)^r$ sets are divided into three groups, each of $3^{r-1}k^r$ sets.

Step 3: From each set in the first group, the smallest rank unit is selected; from each set in the second group; the median rank unit is selected, and from each set in the third group, the largest rank unit is selected. This step yields $(3k)^{r-1}$ sets, $3^{r-2}k^{r-1}$ sets in each group.

Step 4: Without doing any actual quantification, from the $3^{r-2}k^{r-1}$ sets in the first group, smallest rank unit is selected, from the $3^{r-2}k^{r-1}$ sets in the second group; the median rank unit is selected, and from the $3^{r-2}k^{r-1}$ sets in the third group; the largest rank unit is selected. This step yields $(3k)^{r-2}$ sets, $3^{r-3}k^{r-2}$ sets in each group each set of size $3k$.

Step 5: The process is continued using Steps (3) and (4) until we end up with one rth stage balanced groups ranked set sample of size $3k$.

The procedure is repeated n times if needed to get a sample of size $3kn$ from initial $(3k)^{r+1}n$ units. It is clear that, based on MBGRSS method, the measured units at the last stage involves the same number of minimums, medians and maximums, which is different from the usual RSS where we select the ith smallest ranked unit from the ith sample. These features make the MBGRSS more practical in the field since it is easy to identify the minimums or the maximums and the medians of the sample particularly when sample size is odd. Indeed the MBGRSS, which is, based on $(3k)^{r+1}$ units, is more informative than both RSS and SRS, since these later two methods are based on $3k$ and $9k^2$ units respectively. For even and odd sample sizes, the suggested method is denoted by MBGRSSE and MBGRSSO, respectively.

Let us consider the following example to illustrate MBGRSS for estimating the population median.

2. Example

Consider the case of $r=3$ and $k=1$, then $m=3k=3$. So we will select $(3k)^{r+1}=81$ units, which are $X_1, X_2, ..., X_{81}$. Allocate the 81 selected units into $(3k)^r=27$ sets each of size 3. The 3 units of each set are ranked with respect to the variable of interest as follows: $\{X_{i(1:3)}, X_{i(2:3)}, X_{i(3:3)}\}$, $(i=1,2,...,27)$. Now, allocate the 27 sets into 3 groups, each of 9 sets as:

First group: $\{X_{i(1:3)}, X_{i(2:3)}, X_{i(3:3)}\}$, $(i=1,2,...,9)$,

Second group: $\{X_{i(1:3)}, X_{i(2:3)}, X_{i(3:3)}\}$, $(i=10,11,...,18)$,

Third group: $\{X_{i(1:3)}, X_{i(2:3)}, X_{i(3:3)}\}$, $(i=19,20,...,27)$.

Then, for $r=1$, we select the smallest ranked unit from each set in the first group, and the median ranked unit from each set in the second group, and the largest ranked unit from each set in the third group. We then allocate these selected units into 27 sets, 9 sets in each group.

First group: $\{X^{(1)}_{3(i-1)+1(1:3)}, X^{(1)}_{3(i-1)+2(1:3)}, X^{(1)}_{3(i-1)+3(1:3)}\}$, $(i=1,2,3)$

Second group: $\{X^{(1)}_{3(i-1)+1(2:3)}, X^{(1)}_{3(i-1)+2(2:3)}, X^{(1)}_{3(i-1)+3(2:3)}\}$, $(i=4,5,6)$

Third group: $\{X^{(1)}_{3(i-1)+1(3:3)}, X^{(1)}_{3(i-1)+2(3:3)}, X^{(1)}_{3(i-1)+3(3:3)}\}$, $(i=7,8,9)$.

For $r=2$, rank the units within each set in each group,

$$\left\{X^{(2)}_{i(1:3)}, X^{(2)}_{i(2:3)}, X^{(2)}_{i(3:3)}\right\},\ (i=1,2,...,9).$$

Then, we select the smallest ranked unit from each set in the first group, and the median ranked unit from each set in the second group, and the largest ranked unit from each set in the third group. We then allocate these selected units into 9 sets, 3 sets in each group.

First group: $\left\{X^{(2)}_{1(1:3)}, X^{(2)}_{2(1:3)}, X^{(2)}_{3(1:3)}\right\}$,

Second group: $\left\{X^{(2)}_{4(2:3)}, X^{(2)}_{5(2:3)}, X^{(2)}_{6(2:3)}\right\}$,

Third group: $\left\{X^{(2)}_{7(3:3)}, X^{(2)}_{8(3:3)}, X^{(2)}_{9(3:3)}\right\}$.

For $r=3$, rank the units within each set in each group

$$\left\{X^{(3)}_{i(1:3)}, X^{(3)}_{i(2:3)}, X^{(3)}_{i(3:3)}\right\},\ (i=1,2,3).$$

Then, we select the smallest ranked unit from each set in the first group, and the median ranked unit from each set in the second group, and the largest ranked unit from each set in the third group. The median of these units is considered as an estimator of the population median as

$$\hat{\eta}^{(r)}_{MBGRSSO} = \text{median}\left\{X^{(3)}_{1(1:3)}, X^{(3)}_{2(2:3)}, X^{(3)}_{3(3:3)}\right\}$$

It is of interest to note here that for $m=3$ at the first stage, MBGRSS is the same as RSS for estimating the population mean.

3. Estimation of the population median

Let $X_1, X_2, ..., X_m$ be a random sample with pdf $f(x)$, cdf $F(x)$, a finite mean μ and variance σ^2. Let $X_{11h}, X_{12h}, \ldots, X_{1mh}; X_{21h}, X_{22h}, \ldots, X_{2mh}; \ldots; X_{m1h}$, $X_{m2h}, \ldots, X_{mmh}$ be independent random variables all with the same cumulative distribution function $F(x)$ in the hth cycle $(h=1,2,...,n)$. Let $X_{i(1:m)h}, X_{i(2:m)h}, \ldots$, $X_{i(m:m)h}$ be the order statistics of the ith sample $X_{i1h}, X_{i2h}, ..., X_{imh}$, $(i=1,2,...,m)$. Then $X_{1(1:1)h}, X_{2(2:2)h}, \ldots, X_{m(m:m)h}$ denote the measured RSS.

The SRS estimator of the population median η is defined as:

$$\hat{\eta}_{SRS} = \begin{cases} X_{\left(\frac{m+1}{2}:m\right)h} & \text{, if } m \text{ is odd} \\ \frac{1}{2}\left(X_{\left(\frac{m}{2}:m\right)h} + X_{\left(\frac{m+2}{2}:m\right)h}\right) & \text{, if } m \text{ is even.} \end{cases} \quad (1)$$

The RSS estimator of the population median η from a sample of size *m* is given by

$$\hat{\eta}_{RSS} = \text{median}\left\{X_{i(i:m)h}, i = 1, 2, ..., m\right\}. \tag{2}$$

If *m* is odd, in the *h*th cycle $\left(h = 1, 2, ..., n\right)$, let $X^{(r)}_{i(1:m)h}$ be the lowest ranked unit of the *i*th sample $\left(i = 1, 2, ..., k\right)$, $X^{(r)}_{i\left(\frac{m+1}{2}:m\right)h}$ be the median of the *i*th sample $\left(i = k+1,\ k+2, ..., 2k\right)$, and $X^{(r)}_{i(m:m)h}$ be the largest ranked unit of the *i*th sample $\left(i = 2k+1,\quad 2k+2, ..., 3k\right)$. So that $X^{(r)}_{1(1:m)h}, X^{(r)}_{2(1:m)h}, ..., X^{(r)}_{k(1:m)h}, X^{(r)}_{k+1\left(\frac{m+1}{2}:m\right)h}$, $X^{(r)}_{k+2\left(\frac{m+1}{2}:m\right)h}$, $\ldots$, $X^{(r)}_{2k\left(\frac{m+1}{2}:m\right)h}$, $X^{(r)}_{2k+1(m:m)h}, X^{(r)}_{2k+2(m:m)h}, ..., X^{(r)}_{3k(m:m)h}$ denote the measured MBGRSSO. Note that, the first *k* units are iid, and the second *k* units are iid and the last *k* units are iid. However, all units are independent but not identically distributed. The estimator of the population median using MBGRSSO is given by

$$\hat{\eta}^{(r)}_{MBGRSSO} = \text{median}\left\{\begin{array}{ll} X^{(r)}_{i(1:m)h} & , i = 1, ..., k; \\ X^{(r)}_{i\left(\frac{m+1}{2}:m\right)h} & , i = k+1, ..., 2k; \\ X^{(r)}_{i(m:m)h} & , i = 2k+1, ..., 3k \end{array}\right\}. \tag{3}$$

If *m* is even, let $X^{(r)}_{i(1:m)h}$ be the lowest ranked unit of the *i*th sample $\left(i = 1, 2, ..., k\right)$, let $\frac{1}{2}\left(X^{(r)}_{i\left(\frac{m}{2}:m\right)h} + X^{(r)}_{i\left(\frac{m+2}{2}:m\right)h}\right)$ be the median of the *i*th sample $\left(i = k+1, k+2, ..., 2k\right)$, and $X^{(r)}_{i(m:m)h}$ be the largest ranked unit of the *i*th sample $\left(i = 2k+1,\quad 2k+2, ..., 3k\right)$. In this case $X^{(r)}_{1(1:m)h}, ..., X^{(r)}_{k(1:m)h}$, $\frac{1}{2}\left(X^{(r)}_{k+1\left(\frac{m}{2}:m\right)h} + X^{(r)}_{k+1\left(\frac{m+2}{2}:m\right)h}\right), \ldots,$ $\frac{1}{2}\left(X^{(r)}_{2k\left(\frac{m}{2}:m\right)h} + X^{(r)}_{2k\left(\frac{m+2}{2}:m\right)h}\right)$, $X^{(r)}_{2k+1(m:m)h}$, $..., X^{(r)}_{3k(m:m)h}$ denote the measured MBGRSSE. In the case of even sample size, the MBGRSSE estimator is given by:

$$\hat{\eta}^{(r)}_{MBGRSSE} = \text{median}\left\{ \begin{array}{ll} X^{(r)}_{i(1:m)h} & , i = 1,...,k; \\ \frac{1}{2}\left(X^{(r)}_{i\left(\frac{m}{2}:m\right)h} + X^{(r)}_{i\left(\frac{m+2}{2}:m\right)h} \right) & , i = k+1,...,2k; \\ X^{(r)}_{i(m:m)h} & , i = 2k+1,...,3k \end{array} \right\}. \quad (4)$$

4. Simulation Study

In this section, to compare the efficiency of proposed estimators of the population median using MBGRSS method relative to SRS method, seven probability distribution functions were considered for the populations: uniform, normal, beta, logistic, exponential, gamma and weibull. We compare the average of 60,000 sample estimates using $k = 1, 2, 3$ corresponding to the sample sizes $m = 3, 6, 9$ respectively. Without loss of generality for simulation, assume the cycle is repeated once. If the distribution is symmetric, the efficiency of RSS and MGBRSS relative to SRS is defined as:

$$eff\left(\hat{\eta}_{SRS}, \hat{\eta}_{RSS}\right) = \frac{\text{Var}\left(\hat{\eta}_{SRS}\right)}{\text{Var}\left(\hat{\eta}_{RSS}\right)} \text{ and } eff\left(\hat{\eta}_{SRS}, \hat{\eta}^{(r)}_{MBGRSS}\right) = \frac{\text{Var}\left(\hat{\eta}_{SRS}\right)}{\text{Var}\left(\hat{\eta}^{(r)}_{MBGRSS}\right)}.$$

If the distribution is asymmetric, the efficiency is defined as:

$$eff\left(\hat{\eta}_{SRS}, \hat{\eta}_{RSS}\right) = \frac{\text{MSE}\left(\hat{\eta}_{SRS}\right)}{\text{MSE}\left(\hat{\eta}_{RSS}\right)} \text{ and } eff\left(\hat{\eta}_{SRS}, \hat{\eta}^{(r)}_{MBGRSS}\right) = \frac{\text{MSE}\left(\hat{\eta}_{SRS}\right)}{\text{MSE}\left(\hat{\eta}^{(r)}_{MBGRSS}\right)},$$

where the $\text{MSE}\left(\hat{\eta}^{(r)}_{MBGRSS}\right)$ is the mean square error of $\hat{\eta}^{(r)}_{MBGRSS}$, which is given by

$$\text{MSE}\left(\hat{\eta}^{(r)}_{MBGRSSj}\right) = \text{Var}\left(\hat{\eta}^{(r)}_{MBGRSSj}\right) + \left[\text{Bias}\left(\hat{\eta}^{(r)}_{MBGRSSj}\right)\right]^2, \ j = E, O.$$

Results of the efficiency and bias values are given in Tables 1-3 with $m = 3, 6, 9$ for $r = 1, 2, 3$.

Table 1. The efficiency for estimating the population median using RSS and MBGRSS for sample size $m = 3$ and $r = 1, 2, 3$

Distribution		RSS	MBGRSS		
			$r = 1$	$r = 2$	$r = 3$
Uniform (0,1)	*Eff*	1.454	1.454	1.974	3.795
Normal (0,1)	*Eff*	1.613	1.613	2.393	5.102
Beta (4,4)	*Eff*	1.582	1.582	2.303	4.797
Logistic (0,1)	*Eff*	1.718	1.718	2.507	5.523
Exponential (1)	*Eff*	1.787	1.787	2.734	6.212
	Bias	0.086	0.086	0.062	0.027
Gamma (2,1)	*Eff*	1.702	1.702	2.583	5.597
	Bias	0.087	0.087	0.060	0.029
Weibull (1,3)	*Eff*	1.815	1.815	2.726	6.383
	Bias	0.258	0.258	0.174	0.088

Table 2. The efficiency for estimating the population median using RSS and MBGRSS for sample size $m = 6$ and $r = 1, 2, 3$

Distribution		RSS	MBGRSS		
			$r = 1$	$r = 2$	$r = 3$
Uniform (0,1)	*Eff*	2.413	2.131	8.245	37.523
Normal (0,1)	*Eff*	2.730	2.352	9.370	43.084
Beta (4,4)	*Eff*	2.595	2.379	8.977	41.539
Logistic (0,1)	*Eff*	2.786	2.363	9.726	45.404
Exponential (1)	*Eff*	2.872	2.337	9.428	38.932
	Bias	0.054	0.074	0.042	0.032
Gamma (2,1)	*Eff*	2.807	2.447	9.349	41.001
	Bias	0.048	0.073	0.045	0.033
Weibull (1,3)	*Eff*	2.841	2.464	9.311	38.232
	Bias	0.147	0.218	0.130	0.096

Table 3. The efficiency for estimating the population median using RSS and MBGRSS for sample size $m=9$ and $r=1,2,3$

Distribution		RSS	MBGRSS		
			$r=1$	$r=2$	$r=3$
Uniform (0,1)	*Eff*	2.981	2.343	11.621	71.212
Normal (0,1)	*Eff*	2.942	2.482	13.591	82.241
Beta (4,4)	*Eff*	2.772	2.331	13.230	80.107
Logistic (0,1)	*Eff*	2.882	2.479	13.830	85.209
Exponential (1)	*Eff*	3.002	2.478	14.924	90.870
	Bias	0.018	0.022	0.004	0.001
Gamma (2,1)	*Eff*	2.901	2.461	14.323	86.035
	Bias	0.019	0.022	0.004	0.000
Weibull (1,3)	*Eff*	3.046	2.503	14.767	91.905
	Bias	0.050	0.067	0.011	0.002

The results in Tables 1-3, indicate that:

1. Gain in efficiency is obtained using MBGRSS for estimating the population median. As an example, with $m=9$ and $r=3$ for estimating the median of the normal distribution, the efficiency of MBGRSSO is 82.241.
2. MBGRSS estimators are unbiased of the population median when the underlying distribution is symmetric about the population mean.
3. The efficiency of MBGRSS is increasing in r for specific value of the sample size. For example, for $m=6$ and $r=1,2,3$ the efficiency values are 2.131, 8.245 and 37.523 respectively for estimating the median of the uniform distribution.
4. The efficiency of MBGRSS estimators is increasing in the sample size for specific r. As an example, when the underlying distribution is exponential with parameter 1, for $r=3$ and $m=3,6,9$, the efficiency values are 6.212, 38.932 and 90.870 respectively.
5. When the underlying distribution is asymmetric, the small bias of MBGRSS is decreasing in r. For example, when the underlying distribution is gamma with parameters (2,1), for $m=9$ and $r=1,2,3$, the efficiency of MBGRSSO is 2.461, 14.323 and 86.035 with biases 0.022, 0.004 and 0.000, respectively.

5. An Application

In this section, we will investigate the efficiency of MBGRSS method for estimating the population median of 64 olive trees' yields in West Jordan. The data was actually collected via RSS in 1999. The chosen ranker visually judged the olives yield for each set size See Al-Omari (1999). Results are summarized in Tables 4–6.

Table 4. The efficiency of RSS with respect to SRS for estimating the median of olive yields with $m = 3, 6, 9$

	Bias		Efficiency
	SRS	RSS	
$m = 3$	1.26461	1.06925	1.44859
$m = 6$	1.06864	0.832194	2.27847
$m = 9$	0.812175	0.533728	3.79903

Table 5. The efficiency and bias values of MBGRSS with respect to SRS for estimating the median of olive yields with $m = 3$ for $r = 1, 2, 3, 4, 5$

	$r=1$	$r=2$	$r=3$	$r=4$	$r=5$
Bias	1.06925	0.921435	0.657382	0.430258	0.266557
Efficiency	1.44859	2.06416	4.38985	11.9381	40.4151

Table 6. The efficiency and bias values of MBGRSS with respect to SRS for estimating the median of olive yields with $m = 6, 9$ for $r = 1, 2, 3$

	$r=1$	$r=2$	$r=3$
		$m = 6$	
Bias	0.98927	0.764353	0.656441
Efficiency	2.0332	6.01133	13.6648
		$m = 9$	
Bias	0.0666892	0.208683	0.153532
Efficiency	2.50285	32.0339	97.2748

Let w_i, $i = 1, 2, ..., 64$ be the olive yield of the ith tree. The exact median η and variance σ^2 of the population are

$$\eta = 8.1,\ \sigma^2 = \frac{1}{64}\sum_{i=1}^{64}(w_i - \mu)^2 = 26.112 \text{ kg}^2 / \text{tree}.$$

Based on Tables 4 and 5, it can be noted that the MBGRSS is more efficient than the SRS and RSS methods based on the same number of measured units. Also, the efficiency of MBGRSS is increasing as the sample size increasing.

6. Conclusions

Based on the above results we can conclude that the properties of $\hat{\eta}^{(r)}_{MBGRSS}$ are:

1. If the underlying distribution is symmetric about the population mean μ, then
   a. $\hat{\eta}^{(r)}_{MBGRSS}$ is an unbiased estimator of the population median.
   b. $\mathrm{Var}\left(\hat{\eta}^{(r)}_{MBGRSS}\right) < \mathrm{Var}\left(\hat{\eta}_{SRS}\right)$, implies that MBGRSS is more efficient than SRS.
   c. MBGRSS is more efficient than RSS for $r > 1$, since $\mathrm{Var}\left(\hat{\eta}^{(r)}_{MBGRSS}\right) < \mathrm{Var}\left(\hat{\eta}_{RSS}\right)$ for $r > 1$.
2. If the underlying distribution is asymmetric about μ, then $\hat{\eta}^{(r)}_{MBGRSS}$ has a small bias, and the bias decreases in *r*.
3. The efficiency of MBGRSS estimators is increasing as the number of stage increasing.

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